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Vibration Control of Plates Using Discretely Distributed Piezoelectric Quasi-Modal Actuators/Sensors

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A novel approach is presented for vibration control of smart plates using discretely distributed piezoelectric actuators and sensors. The new method consists of techniques for designing quasi-modal sensors and quasi-modal actuators. The modal coordinates and the modal velocities are obtained approximately from the outputs of the discretely distributed piezoelectric sensor elements, whereas the modal actuators are implemented by applying proper voltages on each actuator element. The observation error of the modal sensor is analyzed, and an upper bound for the observation error is determined. The control spillover of the modal actuator is also estimated, and an upper bound of the control spillover is also found. The criteria are developed for finding the optimal locations and sizes of both piezoelectric sensor and actuator elements. In the optimality criteria the optimal locations and sizes of the sensor elements can be found by minimizing the observation error of the modal sensor, and those of the actuator elements can be obtained by minimizing both the control energy and the control spillover. The results obtained using the present optimal criteria show that they do not depend on the initial condition of vibration of the structures, nor do they depend on the control gains.

I. Introduction

WITH the development of the space technology, space structures are becoming larger and more flexible, and, consequently, vibration control of large space structures becomes more important than ever before. In recent years smart materials, such as piezoelectric materials, have been used extensively as distributed sensor and actuator. Because the piezoelectric sensor and actuator are distributed on the host structure, an accurate control can be achieved.

Smart structures have attracted significant attention in the field of control and dynamics, and many achievements have been made in the past 10 years. In 1985 Bailey and Hubbard[1] used polyvinylidene fluoride (PVDF) film as the distributed actuator to perform vibration control of cantilever beam. Since then, mathematical models for the smart beams, plates, and shells integrated with the piezoelectric actuators and sensors have been established.[2–12] Some finite element formulations for the piezoelectric plates and shells are also derived. Based on these models, much research on active vibration control, hybrid control, optimal placement, and sizing of the actuators, as well as experimental investigation, has been studied by many researchers all over the world.[13–21] More details can be found in several review papers.[22–25]

Independent modal space control[26] is an effective method for vibration control of smart structures in which modal sensor and modal actuator are needed. Modal control for the smart plates can be performed using the piezoelectric modal sensor and modal actuator based on reshaping the piezoelectric layers with little control and observation spillover.[14–17] However, these modal sensor and actuator are sensitive to the change of the number of the controlled modes and to the change of the structures as well, and it is hard to suit these changes by software. An alternative modal sensor and modal actuator, which can overcome the preceding drawbacks, was presented based on segmentation of piezoelectric sensor and actuator layers.[27,28] It can suit the changes by software without changing the pattern of the piezoelectric transducers. In this method the modal sensor and modal actuator are designed based on segmenting the piezoelectric laminas into several continuous elements. However, the fully covered or the continuously distributed piezoelectric sensors and actuators are not practical in vibration control of large flexible structures such as the solar arrays in spacecraft. Therefore, other methods for vibration control of large flexible structures are needed in which a structure is not fully covered by the piezoelectric material, and only several parts of the structure are covered by the piezoelectric elements. Moreover, the optimal placement and sizing of the piezoelectric elements should also be considered in order to achieve better control results.

In this paper modal control of smart plates is investigated using discrete piezoelectric sensor and actuator elements. A novel approach for vibration control of smart plates is presented using optimized discretely distributed piezoelectric actuators and sensors. The method includes the approach to design quasi-modal sensor and quasi-modal actuator, the observation spillover and control spillover analysis, and the criteria for finding the optimal locations and sizes of the piezoelectric sensor and actuator elements. The optimal results obtained from the criterion do not depend on the initial condition, nor do they depend on the control gains. Finally, as an illustrative example, vibration control of a simply supported rectangular plate is performed, and the effectiveness of the optimal placement of piezoelectric actuator patches is demonstrated by numerical simulation.

II. Basic Equations

Consider a thin rectangular plate with length $2a$ and width $2b$, on which several rectangular piezoelectric patches are bonded, as shown in Fig. 1. The piezoelectric elements bonded on the upper surface of the plate are used as actuators, and those bonded on the lower surface are used as sensors. Assume that the piezoelectric elements are perfectly bonded, and the effects of the bonding layers on the stiffness and mass of the smart plate can be neglected.

The charge generated by the nth sensor element can be written as[14]

$$q_n(t) = -r_n e_{31n} \iint\limits_{S_{sn}} \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right] \mathrm{d}x\,\mathrm{d}y \tag{1}$$

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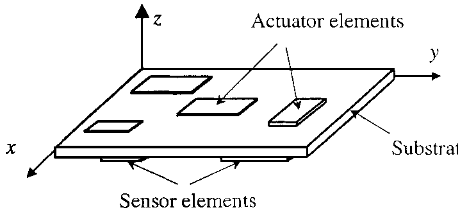

Fig. 1 Plate with piezoelectric elements.

where $q_n(t)$, r_n, and e_{31n} are the charge output, the z coordinate of the midplain, and the piezoelectric stress constant of the nth sensor element, respectively; $w(x, y, t)$ is the transverse displacement of the plate; N_s is the number of the piezoelectric sensor patches; and $S_{\text{sn}}(\tilde{x}_{\text{ln}} \leq x \leq \tilde{x}_{\text{rn}}, \tilde{y}_{\text{bn}} \leq y \leq \tilde{y}_{\text{tn}})$ the area covered by the nth piezoelectric sensor element.

The differential equation of motion of the smart plate with discretely distributed actuator patches can be derived as

$$\rho h \frac{\partial^2 w}{\partial t^2} + D\nabla^4 w = \nabla^2 m(x, y, t) \tag{2}$$

where ρh and D are the equivalent mass density per unit area and the equivalent bending stiffness of the smart plate respectively, and $m(x, y, t)$is the bending moment distribution induced by the actuator elements, which can be expressed as

$$m(x, y, t) = \sum_{n=1}^{N} k_n V_n(t)[H(x - x_{\text{ln}}) - H(x - x_{\text{rn}})][H(y - y_{\text{bn}}) - H(y - y_{\text{tn}})] \tag{3}$$

where $V_n(t)$ is the input voltage applied on the nth actuator patches, k_n is the coupling constants related to the dimensions and parameters of the host plate and the actuator element,[8] $H(.)$ is the Heavside function, N is the number of the actuator patches, and $(x_{\text{ln}}, y_{\text{tn}})$ and $(x_{\text{rn}}, y_{\text{bn}})$ are the coordinates of the top-left corner and the right-bottom corner of the nth actuator element, respectively. Equation (2) is the actuator equation that establishes the relationship between the vibration of the plate and the control voltages exerted on the actuator elements.

III. Quasi-Modal Sensor Design and Observation Spillover Estimate

A. Measurement and Observation of the Modal Coordinates

Transforming the sensor equation (1) into modal space yields

$$q_n(t) = -r_n e_{31n} \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \eta_{ij}(t) \iint_{S_{\text{sn}}} \left(\frac{\partial^2 W_{ij}}{\partial x^2} + \frac{\partial^2 W_{ij}}{\partial y^2} \right) \mathrm{d}x\, \mathrm{d}y \qquad n = 1, 2, \ldots, N_S \tag{4}$$

where $W_{ij}(x, y)$ is the ijth modal function and $\eta_{ij}(t)$ the ijth modal coordinate of the smart plate. For convenience the two modal indices i and j are replaced by one index i in order of increasing magnitude of frequency, then Eq. (4) becomes

$$q_n = \sum_{i=1}^{\infty} \alpha_{\text{ni}} \eta_i(t), \qquad n = 1, 2, \ldots, N_S \tag{5}$$

where

$$\alpha_{\text{ni}} = -r_n e_{31n} \iint_{S_{\text{sn}}} \left(\frac{\partial^2 W_i}{\partial x^2} + \frac{\partial^2 W_i}{\partial y^2} \right) \mathrm{d}x\, \mathrm{d}y, \qquad i = 1, 2, \ldots \tag{6}$$

are the coefficients related to the locations and sizes of the sensor elements and the modal functions of the plate.

Although the plate has an infinite number of modes theoretically, the number of the mode that can be excited by external forces are finite because of the limitation of the excitation energy. For this reason only the first $M (M > Ns)$ modes of the plate are considered, and Eq. (5) becomes

$$\{q(t)\} = [A_1]\{\eta_1(t)\} + [A_2]\{\eta_2(t)\} \tag{7}$$

where $\{q(t)\} \in R^{N_S}$ is a vector of output charges generated by the sensor elements, $[A_1] \in R^{N_S \times N_S}$ and $[A_2] \in R^{N_S \times (M - N_S)}$ are related to the output charges of the sensor elements and the modal coordinates, $\{\eta_1(t)\} \in R^{N_S}$ and $\{\eta_2(t)\} \in R^{M - N_S}$ the vectors composed of the first N_S modal coordinates and the residual $M - N_S$ modal coordinates, respectively.

Matrix $[A_1]$ can be made nonsingular as long as the piezoelectric elements are placed properly (see Sec. VI.A), and then we have

$$\{\eta_1(t)\} = [A_1]^{-1}\{q(t)\} - [A_1]^{-1}[A_2]\{\eta_2(t)\} \tag{8}$$

Neglecting the second term in the right terms, the first N_S modal coordinates can be estimated approximately by

$$\{\eta_1^*(t)\} = [A_1]^{-1}\{q(t)\} \tag{9}$$

where $\{\eta_1^*(t)\}$ is the vector containing the N_S observed modal coordinates. The error between the observed and the real modal coordinates can be expressed as

$$\{\Delta(t)\} = \{\eta_1^*(t)\} - \{\eta_1(t)\} = [A_1]^{-1}[A_2]\{\eta_2(t)\} \tag{10}$$

Equation (10) indicates that the observed modal coordinates obtained from Eq. (9) are the real ones mixed with the residual modal coordinates with higher frequencies. Therefore, Eq. (9) is called quasi-modal sensors. To ensure the stability of the closed-loop system, the residual components with higher frequency should be filtered out from the observed modal coordinates by low-pass filters. In Sec. V a compensator is used for each controlled mode to remove the residual modes.

B. Observation Spillover Estimate

To minimize the observation error of the quasi-modal sensor, the observation error (observation spillover) will be estimated. From Eq. (10) the norm of the observation error can be expressed as

$$\|\Delta(t)\|^2 = \{\Delta(t)\}^T\{\Delta(t)\} = \{\eta_2(t)\}^T[A_{\text{sp}}]\{\eta_2(t)\} \tag{11}$$

where $[A_{\text{sp}}] = [A_2]^T([A_1]^{-1})^T[A_1]^{-1}[A_2] \in R^{(M - N_S)\times(M - N_S)}$ is a symmetric positive definite matrix. It can be proved that the following inequalities hold:

$$\lambda_{\min}([A_{\text{sp}}]) \leq \frac{\{x\}^T[A_{\text{sp}}]\{x\}}{\{x\}^T\{x\}} \leq \lambda_{\max}([A_{\text{sp}}]) \tag{12}$$

where $\lambda_{\min}([A_{\text{sp}}])$ and $\lambda_{\max}([A_{\text{sp}}])$ are the minimum eigenvalue and maximum eigenvalue of the matrix $[A_{\text{sp}}]$, respectively.

Employing Eqs. (11) and (12), we find

$$\|\Delta(t)\|^2 \leq \lambda_{\max}([A_{\text{sp}}])\|\eta_2(t)\|^2 \tag{13}$$

It is clear that Eq. (13) gives an upper bound of the observation error when Eq. (9) is used to estimate the modal coordinates of the plate. It can also be used to find the optimal locations and sizes of the sensor elements as discussed in Sec. VI.

IV. Quasi-Modal Actuator Design and Control Spillover Estimate

A. Quasi-Modal Actuator Design

To design the modal actuator by using the discretely distributed piezoelectric actuator elements, employing the mode orthogonality, the modal equations of motion for a smart plate can be written as

$$\ddot{\eta}_i(t) + \omega_i^2 \eta_i(t) = \sum_{n=1}^{N} \beta_{\text{in}} V_n(t), \qquad i = 1, 2, \ldots \tag{14}$$

where ω_i^2 is the ith natural frequency of the plate and

$$\beta_{\mathrm{in}} = k_n \int_{y_{\mathrm{bn}}}^{y_{\mathrm{tn}}} \left[\frac{\partial}{\partial x} W_i(x_{\mathrm{ln}}, y) - \frac{\partial}{\partial x} W_i(x_{\mathrm{rn}}, y) \right] \mathrm{d}y + k_n \int_{x_{\mathrm{ln}}}^{x_{\mathrm{rn}}} \left[\frac{\partial}{\partial y} W_i(x, y_{\mathrm{bn}}) - \frac{\partial}{\partial y} W_i(x, y_{\mathrm{tn}}) \right] \mathrm{d}x \tag{15}$$

are called the modal influence coefficients, which indicates the influence of the nth actuator elements on the ith mode. The modal influence coefficients depend on not only the modal shape functions but also the locations and sizes of the actuator elements.

Similarly consider only the first M modes, and write Eq. (14) in the following matrix form:

$$\{\ddot{\eta}_c(t)\} + \left[\Omega_c^2\right]\{\eta_c(t)\} = [B_c]\{V(t)\}$$

$$\{\ddot{\eta}_r(t)\} + \left[\Omega_r^2\right]\{\eta_r(t)\} = [B_r]\{V(t)\} \tag{16}$$

where $\{\eta_c(t)\} \in R^N$ is the vector of N controlled modal coordinates, $\{\eta_r(t)\} \in R^{M-N}$ the vector of $M - N$ uncontrolled (residual) modal coordinates, $[\Omega_c^2] = \mathrm{diag}(\omega_1^2, \omega_2^2, \ldots, \omega_N^2)$, $[\Omega_r^2] = \mathrm{diag}(\omega_{N+1}^2, \omega_{N+2}^2, \ldots, \omega_M^2)$, $\{V(t)\} \in R^N$ the vector composed of the voltages applied on the N piezoelectric actuator elements, and $[B_c]$ and $[B_r]$ are the modal influence matrices.

The modal actuator can actuate the designated modes without affecting the rest modes. If the modal forces for the N controlled modes are designated to be $\{f_c(t)\} = [f_1(t), f_2(t), \ldots, f_N(t)]^T$, then the voltages that can generate such N modal forces satisfy the following equations:

$$[B_c]\{V(t)\} = \{f_c(t)\} \tag{17}$$

If the actuators are distributed properly, the matrix $[B_c]$ is nonsingular, and therefore the voltages applied on the actuators can be obtained as

$$\{V(t)\} = [B_c]^{-1}\{f_c(t)\} \tag{18}$$

Equation (18) gives the voltages needed to generate the designated modal forces according to some selected control laws.

Although the modal actuator can generate a designated modal force for each controlled mode without affecting other controlled modes, it may affect the residual modes. In other words, the control spillover exists, and therefore, Eq. (18) is called the quasi-modal actuator. However, the control spillover will not result in an unstable closed-loop system.[26]

B. Control Spillover Estimate

Denote the forces excited on the residual modes as $\{f_r(t)\}$, which is generated by the control voltage. Substituting Eq. (18) into the second equation of Eq. (16) yields

$$\{f_r(t)\} = [B_r][B_c]^{-1}\{f_c(t)\} \tag{19}$$

which establishes the relationship between the modal control forces and the spillover modal forces. To estimate the degree of the control spillover, the norm of the spillover modal forces can be obtained as

$$\|f_r(t)\|^2 = \{f_r(t)\}^T\{f_r(t)\} = \{f_c(t)\}^T[B_{\mathrm{sp}}]\{f_c(t)\} \tag{20}$$

where $B_{\mathrm{sp}} = ([B_c]^{-1})^T[B_r]^T[B_r][B_c]^{-1}$ is a symmetric positive definite matrix. Employing Eq. (12), we have

$$\|f_r(t)\|^2 \le \lambda_{\max}([B_{\mathrm{sp}}])\|f_c(t)\|^2 \tag{21}$$

Equation (21) establishes the relationship between the control energy and the spillover energy, and it also gives an upper bound of the control spillover.

V. Modal Control of Smart Plate

To perform the modal control of a plate, the modal coordinates and modal velocities are needed. However, the modal coordinates observed by the quasi-modal sensor cannot be used directly because the observation spillover may destabilize the closed system. To solve this problem, the positive position feedback control method[20] is used to control the first N modes of the smart plate independently. For each mode to be controlled, a compensator is designed as follows:

$$\ddot{\xi}_i(t) + 2\zeta_{\mathrm{ci}}\omega_i\dot{\xi}_i(t) + \omega_i^2\xi_i(t) = \omega_i^2\eta_i^*(t), \qquad i = 1, 2, \ldots, N \tag{22}$$

where $\ddot{\xi}_i(t)$ is the response of the compensator to the estimated modal coordinate $\eta_i^*(t)$, and ζ_{ci} the damping ratio of the compensator. The natural frequency of the compensator is set to be equal to the related natural frequency of the smart plate in Eq. (22). The control forces for the first N modes are simply designed as

$$f_i(t) = g_i\xi_i(t) \tag{23}$$

where $g_i \ge 0$ is the control gain.

The compensators in Eq. (22) function as a low-pass filter, which can remove the components with frequencies higher than ω_i from the estimated modal coordinate η_i^* and only the signal with frequency ω_i can pass through. Therefore, the observation spillover can be suppressed significantly by using the compensators.

Another role of the compensator is to change the phase of the modal coordinate such that the response $\xi_i(t)$ of the compensator is 180 deg out of the ith modal velocity $\dot{\eta}_i(t)$. As a result, the control law in Eq. (23) can provide the active damping to the controlled modes, and, therefore, the vibration of the smart plate can be controlled actively by the discretely distributed piezoelectric elements.

VI. Optimal Placement and Sizing of Piezoelectric Actuator and Sensor Elements

It is important to find the optimal locations and sizes of the piezoelectric sensor and actuator elements in vibration control of smart plates because the control effect is highly dependent on the placements of the piezoelectric elements. In this section the optimal placement and sizing of the piezoelectric elements are discussed.

A. Optimal Placement and Sizing of Sensor Elements

The most important issue is how to minimize the observation spillover when the optimal placement and sizing of the sensor is considered. To minimize the observation spillover for any $\{\eta_2(t)\}$ in Eq. (13), the maximum eigenvalue of the matrix $[A_{\mathrm{sp}}]$ should be minimized through optimal placement of the sensor elements, i.e.,

$$J_S = \min_{S_{\mathrm{Sn}}(n = 1, 2, \ldots N_S)} \lambda_{\max}([A_{\mathrm{sp}}]) \tag{24}$$

Equation (24) is the criterion for finding the optimal locations and sizes of the sensor elements.

B. Optimal Placement and Sizing of Actuator Elements

To find the optimal locations and sizes of actuator elements, the following two aspects are considered. One is using less control energy to produce the modal forces needed in the active control process. To this end, consider the relationship between the modal control forces and the control voltages obtained from Eq. (18). The relation between the modal control force vector $\{f_c(t)\}$ and the control voltage vector $\{V(t)\}$ can be obtained from Eq. (18) as

$$\{f_c(t)\}^T\{f_c(t)\} = \{V(t)\}^T[B_c]^T[B_c]\{V(t)\} \tag{25}$$

where $[B_c]^T[B_c]$ is a symmetric square positive definite matrix. Employing Eq. (12) and denoting $[B_{\mathrm{en}}] = [B_c]^T[B_c]$ yields

$$\|V(t)\|^2 = \{V(t)\}^T\{V(t)\} \le \|f_c(t)\|^2 \big/ \lambda_{\min}([B_{\mathrm{en}}]) \tag{26}$$

It can be seen from inequality (26) that for any given modal control forces $\{f_c(t)\}$ the larger the minimum eigenvalue of $[B_{\mathrm{en}}]$ is, the smaller the magnitude of the needed control voltage is.

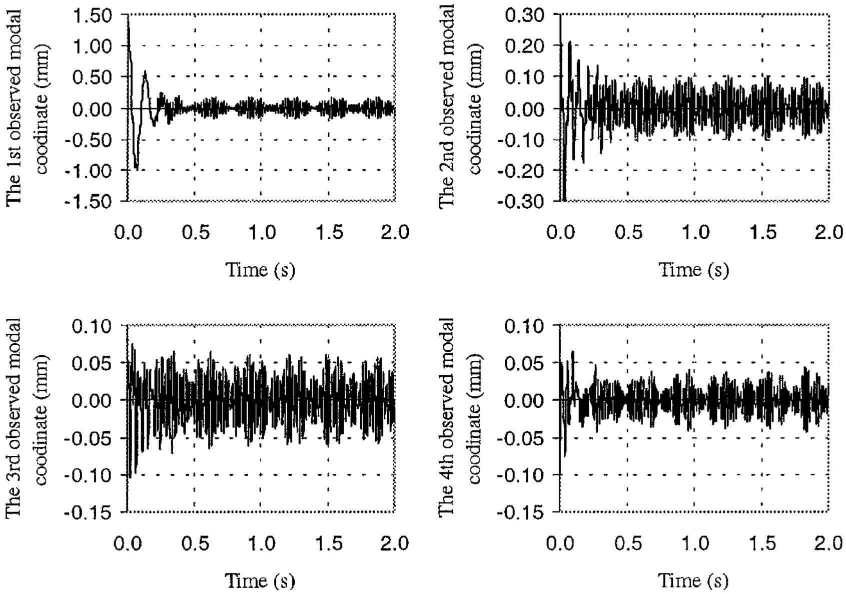

Fig. 2 First four modal coordinates observed by the nonoptimal sensor elements.

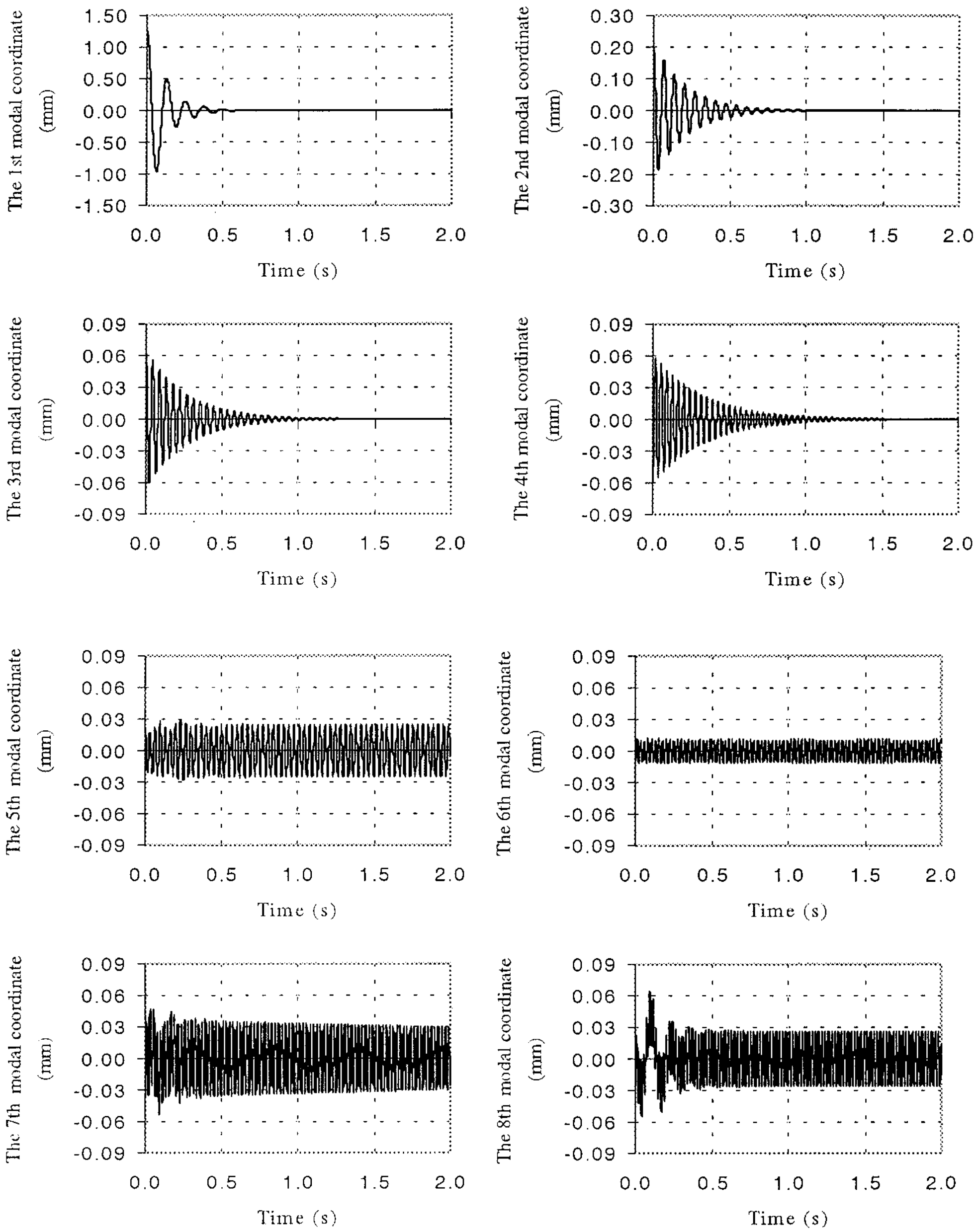

Fig. 3 Four controlled model coordinates and the first four residual modes.

On the other hand, we also expect that the control spillover be as low as possible. Combining these two aspects and noting Eq. (21), the following criterion can be developed:

$$J_a = \min_{S_{\mathrm{an}}\,(n\,=\,1,\,2,\,\ldots N)} \{R_1 \lambda_{\max}([B_{\mathrm{sp}}]) - R_2 \lambda_{\min}([B_{\mathrm{en}}])\} \tag{27}$$

where R_1 and R_2 are the weighting coefficients. In general, because the minimum eigenvalue of the matrix $[B_{\mathrm{en}}]$ is much smaller than the maximum eigenvalue of the matrix $[B_{\mathrm{sp}}]$, R_1 should be selected much larger than R_2 in the optimization process.

In practice, some constraints to the sizes and locations of piezoelectric patches may be imposed in the optimization process.

VII. Illustrative Example

As an illustrative example, consider a simply supported rectangular plate onto which four piezoceramic actuator elements and six PVDF sensor elements are bonded. All of the actuator elements are made of the same lead zirconate titanate (PZT) material, and all of the sensor elements are made of the same PVDF material too. The material properties and dimensions of the system are shown in Table 1. In this example, the six PVDF sensor elements and four PZT actuator elements are used to perform the vibration control of the smart plate. The effects of the piezoelectric elements on the mass density and the bending stiffness are not considered because of their small sizes and light weights.

First of all, the locations and sizes of the six sensor elements are optimized using criterion (24). Each sensor element is assumed to be not larger than 6×4 cm and not smaller than 2×2 cm. Because it is time-consuming to find the global minimum for this optimization problem, the satisfactory local minimum is given instead of the global one. The obtained satisfactory locations and sizes are listed in Table 2 when the first 10 modes are considered ($M = 10$).

Unlike the sensor elements, the dimensions of the four actuator elements are given as $a1 = 0.04$ m, $b1 = 0.015$ m; $a2 = 0.03$ m, $b2 = 0.0125$ m; $a3 = 0.02$ m, $b3 = 0.05$ m; $a_4 = 0.025$ m, $b_4 = 0.025$ m. Therefore, the optimization work is to determine the optimal locations of the actuator elements. In the simulation the weighting coefficients are chosen as $R1 = 10^5/c_n^2$, $R2 = 0.1$, $Q = 10^5$. The obtained satisfactory locations of the four actuators are (0.6315, 0.3926), (0.6295, 0.2095), (0.3398, 0.3428), and (0.7257, 0.3582), respectively.

With the optimum placement of the sensor and actuator elements, one can control the vibration of the plate caused by the sudden removal of a force of $3.5N$ acted on the point (0.5, 0.4). Only the first four modes are controlled, and the damping ratios of the compensators are all taken to be 0.5. The control gains are chosen as $g_1 = 600$, $g_2 = 700$, $g_3 = 1000$, and $g_4 = 1000$.

Table 1 Material and dimensional parameters

Item	Plate	Actuator	Sensor
Mass density, kg/m^3	8000	7600	1780
Young's modulus, GPa	210	63	2
Poisson's ratio	0.3	0.3	0.3
Piezo-constant d_{31}, m/V	——	370×10^{-12}	30×10^{-12}
Thickness, m	0.001	0.0004	0.0001
Length, m	1.0	——	——
Width, m	0.7	——	——

Table 2 Optimum locations and sizes of the sensor elements

Sensor element number	Central coordinate, m	Length, cm	Width, cm
1	(0.7527, 0.2404)	3.45	3.27
2	(0.4926, 0.2315)	5.80	2.73
3	(0.2500, 0.4690)	4.42	3.67
4	(0.2498, 0.2471)	5.54	2.51
5	(0.4882, 0.4788)	2.58	4.00
6	(0.7414, 0.4671)	4.69	3.92

If the locations and sizes of six piezoelectric sensor elements are not optimized, for instance, all six sensor elements with the same size of 4×2 cm are randomly bonded on (0.65, 0.2), (0.45, 0.35), (0.35, 0.45), (0.25, 0.25), (0.50, 0.45), and (0.75, 0.4), respectively; the first four modal coordinates observed by them are shown in Fig. 2. It can be seen that the estimated modal coordinates are obviously different from the exact ones shown in Fig. 3. However, although there exist the obvious observation spillover in the observed modal coordinates, the compensator can remove the components with higher frequencies, and, therefore, satisfactory control results can still be achieved. The accuracy of the estimated modal coordinates can be improved significantly by using the optimal locations and sizes of the sensor elements, as shown in Fig. 4. Figure 5 shows that the remaining high-frequency components in the estimated modal coordinates have been further removed so that the gain margin is enlarged when the compensators are used.

The displacement of the center of the controlled plate is shown in Fig. 6, which shows that the vibration of the plate is suppressed effectively within 1 s. The four controlled modes and the first four residual modes are shown in Fig. 3. It can be seen from Fig. 3 that all

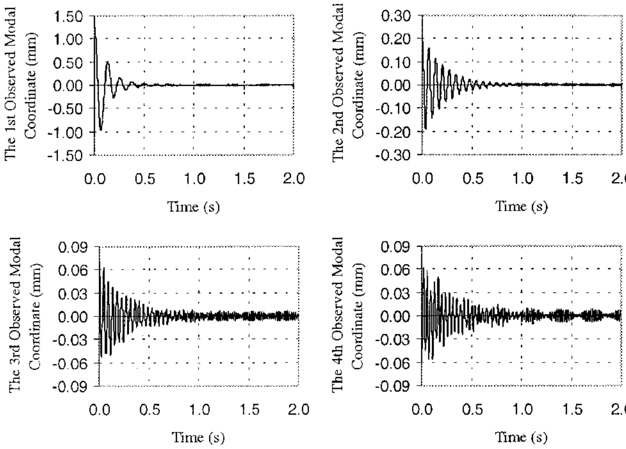

Fig. 4 First four observed modal coordinates by the optimal placed sensor elements.

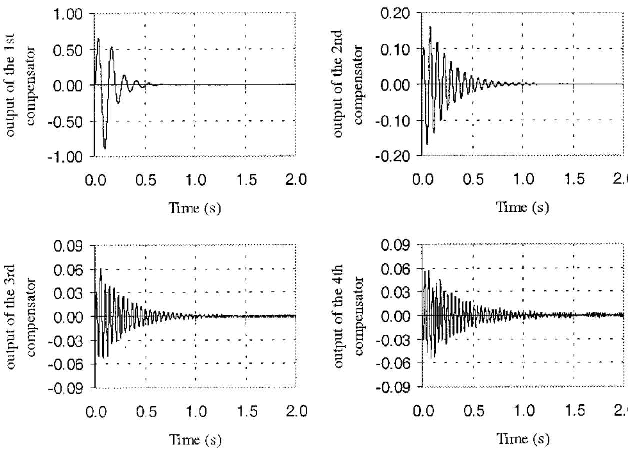

Fig. 5 Outputs of the compensators.

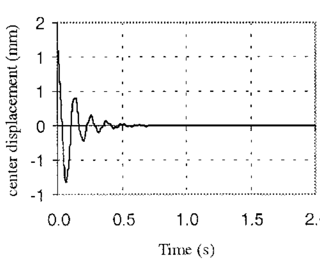

Fig. 6 Center displacement of the plate.

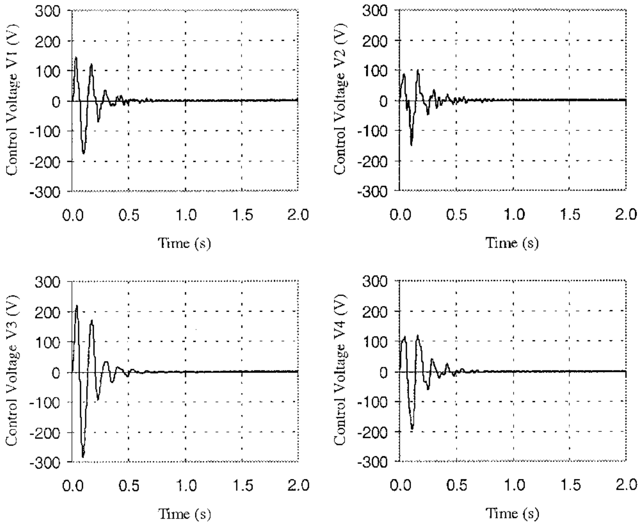

Fig. 7 Control voltage distribution on the optimal placed actuator elements.

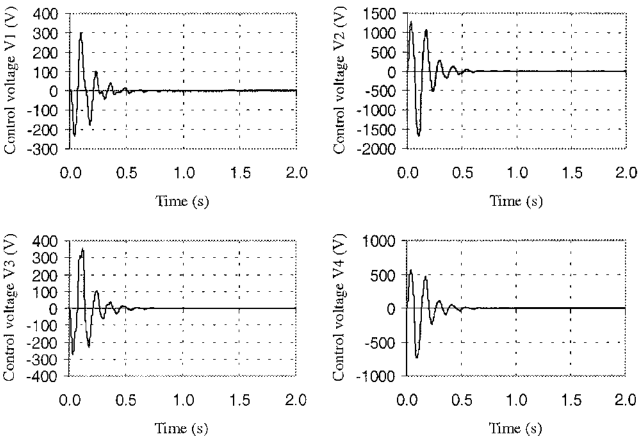

Fig. 8 Control voltage distribution for the non-optimal actuator elements.

the residual modes (especially the eighth mode) are affected by the control voltage. However, the vibration amplitudes of the residual modes are very small, which can be dissipated even by a small damping of the structure. The control voltage distribution of the optimally placed actuator elements is shown in Fig. 7, in which the peak values of the control voltages are less than 300 V. To generate a given modal control force, the needed control voltage distribution on the actuator patches is highly dependent on the placement of the actuators. For a nonoptimal case in which, say, the four actuator patches are located on (0.4, 0.2), (0.4, 0.4), (0.5, 0.35), (0.65, 0.2) respectively, to generate the same modal control forces as the optimal case, the control voltage distribution is shown in Fig. 8. Clearly, the control voltages are significantly decreased by optimizing the locations of the actuator patches.

VIII. Conclusions

In this paper a new approach is presented for vibration control of plate using discretely distributed piezoelectric elements. In this approach the quasi-modal sensor and the quasi-modal actuator are designed to estimate the main modal coordinates of the smart plate and to generate the designated modal control forces, respectively. To reduce the observation spillover, a criterion for finding optimal locations and sizes of the sensor elements is given based on the observation spillover analysis. Similarly, a criterion for optimizing the locations and sizes of the actuator elements is also given in which both the energy and control spillover are considered. In addition, a compensator is used in each mode to perform the modal control of the smart plate independently, which can further suppress the observation spillover so that stability of the active control can be warranted. The simulation results show that the vibration of the plate can be suppressed effectively by using the presented approach.

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